J. Fluid Mech. (1964), *vol.* 20, *part* 2, *pp.* 201–208
Printed in Great Britain

The generation of symmetrical duct velocity profiles of high uniform shear

By J. L. LIVESEY
Engineering Department, University of Manchester
AND J. T. TURNER
Rolls Royce Ltd.

(Received 2 March 1964)

The logical design method of Owen & Zienkiewicz (1957), for the generation of linear shear flow, is successfully applied to the case of symmetrical velocity profiles in a two-dimensional duct. Shear parameters as high as 0·8 (corresponding to $\hat{V}/\bar{V} = 1{\cdot}4$) are obtained experimentally. Some slight modification to the theoretical grid spacing is required because of the non-uniformity of the upstream flow.

The decay of the profiles is confined to that due to the developing boundary layer and to the breakdown of the physically impossible condition at the centre, where symmetry implies a change in sign of the uniform shear.

1. Introduction

During an investigation into the effect of velocity profile shape on the production of secondary flows by cylindrical struts, it became necessary to generate artificial velocity profiles in a two-dimensional wind-tunnel.

Several methods of profile generation are available. In general these are crude in operation, consisting of graded obstructions to the flow in the form of spoiler plates, circular disks with varying sized holes (Livesey, Parker & Jones 1956) or similar devices. These methods suffer from difficulties of assembly and the necessity for tedious experiment before the required profile is produced.

A much more refined method of velocity profile generation is that due to Owen & Zienkiewicz (1957). Here the case of an array of parallel circular rods placed normal to a fluid flow is considered and use is made of known relationships for the flow through gauzes to derive the resistance grading needed to produce a uniform shear flow.

The arrangement of the grid and co-ordinate system is shown in figure 1. The wind tunnel is treated as a long channel with walls at $y = 0, h$ and the grid in the plane $x = 0$. A uniform flow velocity $\bar{V}$ is then transformed into a linear shear $u = \bar{V} + \lambda(y - \frac{1}{2}h)$, where $\lambda h/\bar{V}$ is small so that the streamline deviation is also small. A stream function is introduced which includes a perturbation due to the grid. Then, using some experimental results for fluid flow through a wire gauze, an expression for the spacing of the rods across the tunnel height is obtained. Equation (16) (equation numbers refer to Owen & Zienkiewicz 1957) is

$$\frac{\xi}{1-\xi^2} = K_0\left[1 - \frac{2\lambda h}{\bar{V}}\left(\frac{1}{K_0} + \frac{1}{1+a}\right)\left(\frac{y}{h} - \frac{1}{2}\right)\right],$$

where $$\xi = \frac{d}{s} = \frac{\text{wire diameter}}{\text{spacing}}$$

The values of $(\lambda h/\bar{V})_{\text{max.}}$, restricted by the condition that ξ must be greater than zero at the boundaries, are given in table 1, with an extension to the values in the original paper.

Elder (1959) shows that the case considered by Owen & Zienkiewicz is only a particular case of a more general theory. By means of a shaped wire grid, any flow velocity profile may be changed into the required one.

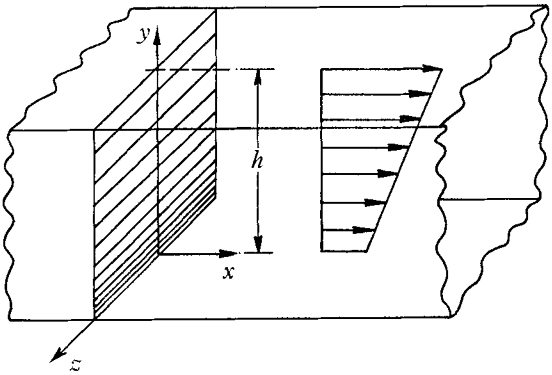


FIGURE 1. Shear flow generator—arrangement of the grid and co-ordinate system.

K_0	0·2	0·4	0·6	0·8	1·0	1·2	1·4	1·6	1·8	2·0
$(\lambda h/\bar{V})_{\text{max.}}$	0·18	0·33	0·45	0·56	0·64	0·71	0·78	0·82	0·86	0·90

TABLE 1

2. Calculation of wire spacing

An iterative method of solution, assuming that the function ξ varies linearly between adjacent wires, was used to determine the wire spacing in the grid. The two equations to be solved are $\xi = d/s$ and equation (16) above. The solution of the second equation is plotted in figure 2 for three values of the shear parameter $(\lambda h)/(\bar{V})$, corresponding to the velocity profile peakiness values $\hat{V}/\bar{V} = 1{\cdot}3$, 1·4 and 1·5.

Using these relationships, the theoretical grid spacing was calculated and was found to give a good approximation to the profiles required. The spacing needed modification over small regions near the centre-line of the tunnel and at the walls.

3. Experimental arrangement

The wind-tunnel used for the experiments was of 21·5 in. × 2 in. rectangular cross-section, in an attempt to ensure that the flow was two-dimensional in the 2 in. vertical plane. Pressure measurements could be taken across the 2 in. height

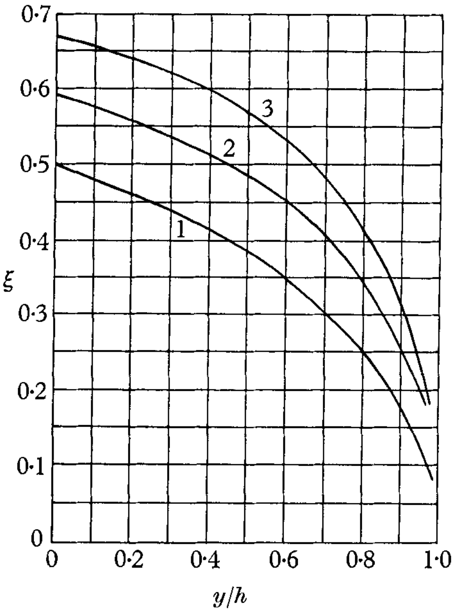


FIGURE 2. Calculated grid wire spacing.

Curve	1	2	3
$\hat{V}/\bar{V}$	1·3	1·4	1·5
K_0	1·0	1·8	3·0
$\lambda h/\bar{V}$	0·6	0·8	1·0

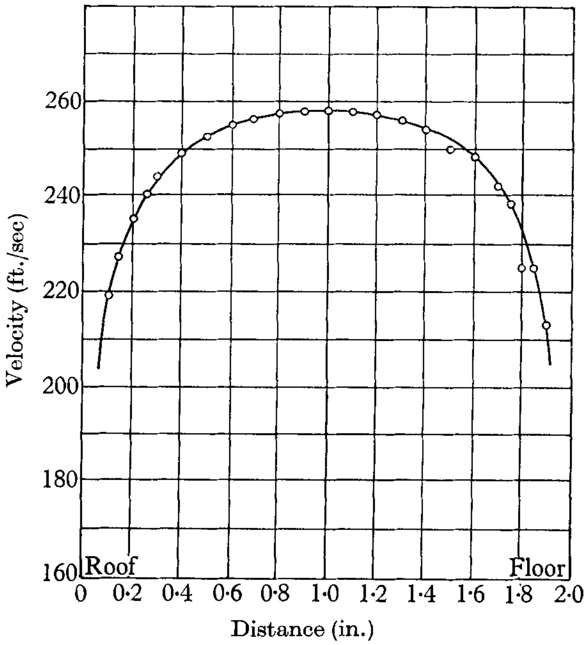


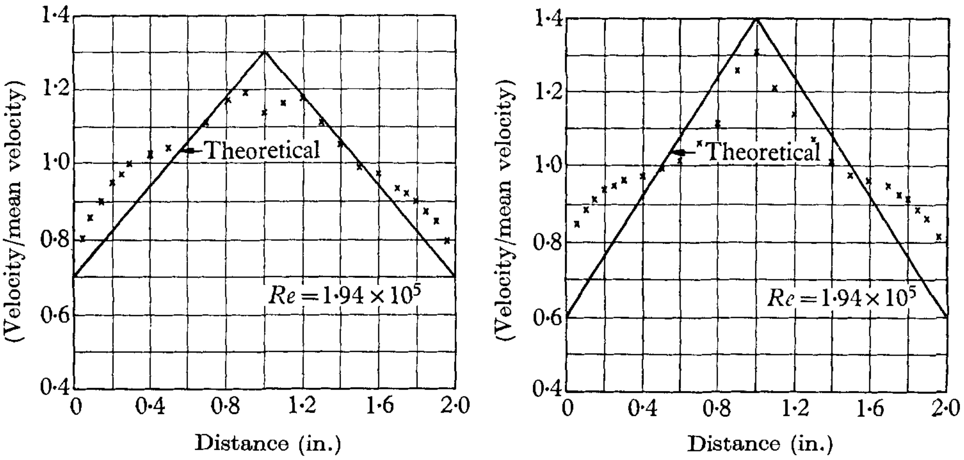
FIGURE 4. Velocity profile, open tunnel $\hat{V}/\bar{V} = 1{\cdot}06$.

of the tunnel at several distances away from the grid, enabling a determination of the stability of the generated profile to be made.

The velocity profile generator consisted of a rigid steel frame about the tunnel, across which were stretched tensioned steel wires. The wires were spaced vertically across the 2 in. tunnel height and kept securely in position by means of grooved steel rollers. The grid arrangement is shown in figures 3(*a*) and (*b*), plate 1; the figures show clearly the method of wire fixing employed. The velocity profile of the flow in the open tunnel at the plane of the wire grid is shown in figure 4.

4. Generation of velocity profiles

An attempt was made to generate symmetrical, linear shear velocity profiles with the extremely high peakiness values of $\hat{V}/\bar{V} = 1{\cdot}3$ and 1·4. Two wire sizes (0·018 in. and 0·036 in. diameter) were investigated in the grid production. It was found possible to achieve the same performance with either size and the larger diameter wire was used in order to reduce the overall work required.



(*a*) (*b*)

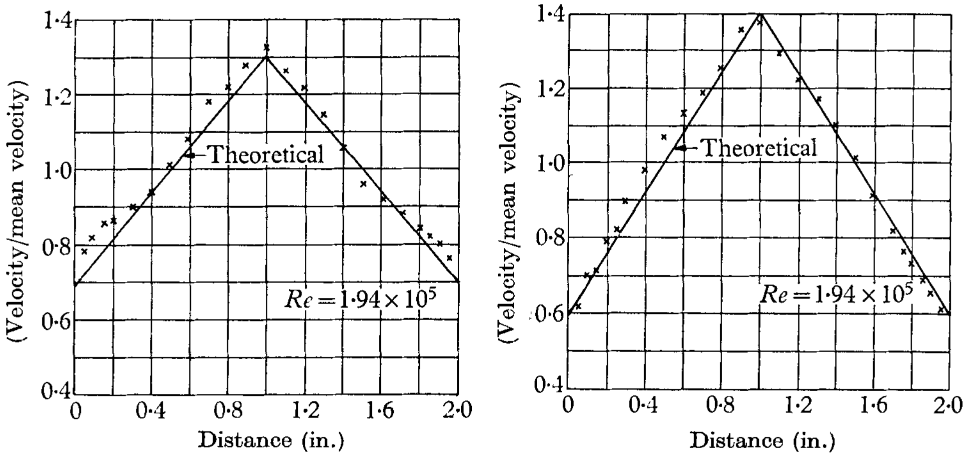
FIGURE 5. Velocity profiles as calculated; 0·0386 in. diameter wire; measured 2 in. downstream from grid.

The experimental velocity profiles obtained are shown, in figures 5 and 7(*a*), as points on the theoretical straight line profiles for the cases attempted. It will be seen that the agreement is very good, apart from the deviation from theory at the tunnel walls. These departures from theory are due to the lower total head of the fluid in the boundary layers upstream of the grid. These regions, therefore, experience a smaller retardation than does the free-stream fluid. It was found to be a relatively easy process to correct these departures from theory, the corrected profiles being shown in figures 6 and 7(*b*). The method of correction is described briefly below.

5. Modifications to grid spacing

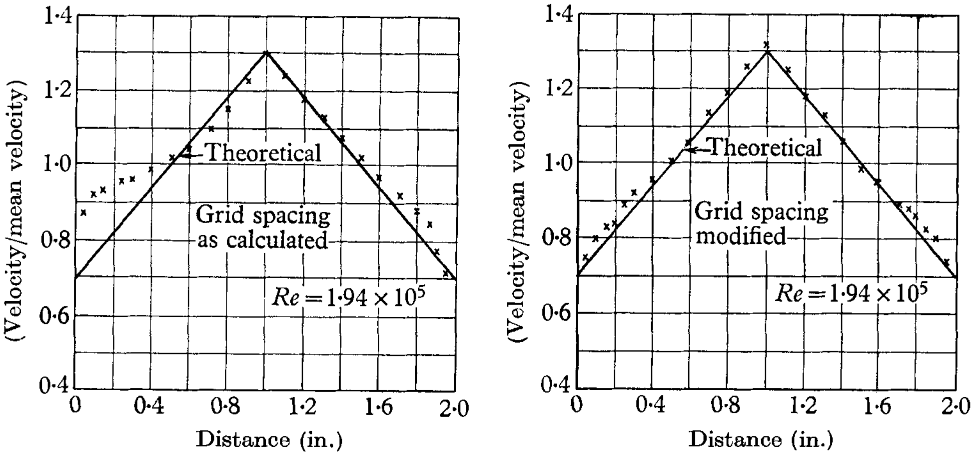
The method of correction was empirical in that no attempt was made to calculate the change in spacing implied by the upstream flow velocity profile.

In the region where the generated velocity profile showed a pronounced increase in velocity (near the walls), the spaces between the wires were reduced to a constant fraction of their calculated value and additional wires were inserted. The success of these crude modifications made it unnecessary to attempt to calculate the variation in spacing implied by the upstream profile—this is possible according to Elder (1959). An example of the calculated and modified wire positions is given in table 2.



(*a*) (*b*)

FIGURE 6. Velocity profiles for modified spacing; 0·0386 in. diameter wire; measured 2 in. downstream from grid.



(*a*) (*b*)

FIGURE 7. Velocity profiles; 0·018 in. diameter wire: (*a*) as calculated; (*b*) modified spacing.

$\hat{V}/\bar{V} = 1{\cdot}3$; wire diameter $= 0{\cdot}0386$ in.

Calculated position (in.)	modified position (in.)
0·0787	0·0675
0·1602	0·1373
0·2446	0·2096
0·3325	0·2850
0·4250	0·3643
0·5249	0·4499
0·6333	0·5428
0·7598	0·6693
0·9438	0·8533

10th wire omitted as beyond centre line.

TABLE 2

6. The decay of generated velocity profiles. Stability

A basic property of artifically produced flow profiles (by which is meant those produced other than by normal boundary-layer growth along the duct walls) is their inherent lack of stability. Thus the flow profile always tends to become more like the naturally developed profile and, where the duct length is sufficient, the generated profile will eventually approximate to the turbulent fully developed profile. Generally this process implies that the velocity profile peakiness ($\hat{V}/\bar{V}$) of the flow will tend to decrease and the profile becomes flatter. This is known as profile decay.

The decay of the profiles produced by the wire grid arrangement is shown in figure 8, plotting the peakiness of the profile ($\hat{V}/\bar{V}$) against the distance of the measuring station downstream from the grid (in tunnel heights $x/2h$). In order to show the changes in velocity profile shape which are not accounted for by the parameter ($\hat{V}/\bar{V}$), the flow profiles obtained at three measuring stations along the tunnel are plotted for comparison in figure 9. This shows that the main changes in shape occur at the centre peak, where the physically impossible condition of a change in sign of the shear is implied by the symmetry of the profile. At the tunnel walls the boundary layer encroaches. Generally the slope of the profile is maintained remarkably well.

The production of velocity profiles by means of spoiler plates and the like, which by their nature introduce non-uniformities in turbulence and large losses into the flow system, has been found to lead to very rapid rates of decay. In comparison with these methods, the profiles produced by the wire grids have the lowest decay rates. This is consequent on their linearity, and approximately constant eddy viscosity.

7. Conclusions

The method of Owen & Zienkiewicz for the generation of linear shear flow has been successfully applied to the generation of symmetrical velocity profiles in a two-dimensional duct. High peakiness profiles ($\hat{V}/\bar{V} = 1{\cdot}4$) are easily ob-

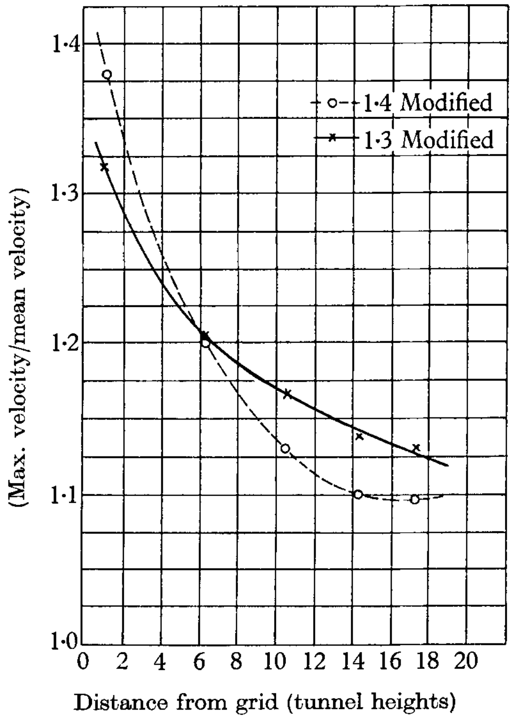


FIGURE 8. Decay of generated profiles.

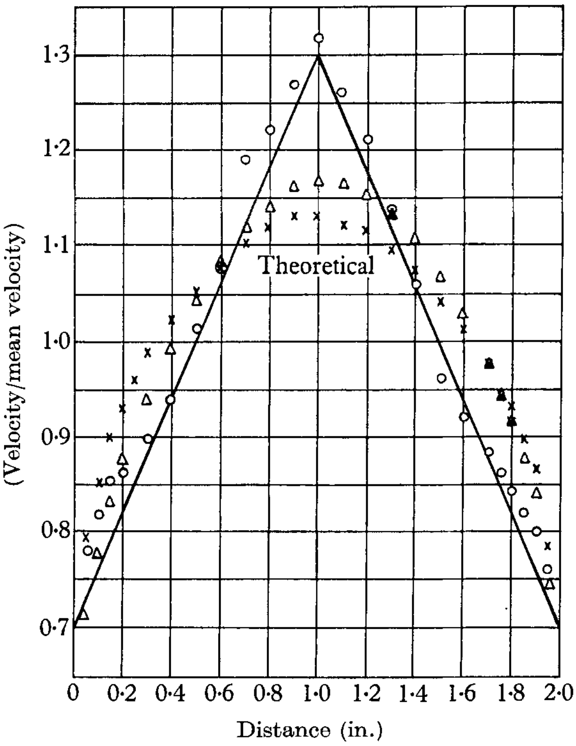


FIGURE 9. Stability of velocity profile; 0·0386 in. wire: ○, $x/2h = 1{\cdot}0$; △, $x/2h = 10{\cdot}0$; ×, $x/2h = 17{\cdot}0$.

tained. Some modification to the theoretical resistance grading of the generating rods is necessary because of the non-uniformity of the upstream flow. The rate of decay of the profiles produced is lower than has been found previously using other methods of generation.

REFERENCES

ELDER, J. W. 1959 *J. Fluid Mech.* **5**, 355–368.
LIVESEY, J. L., PARKER, E. & JONES, P. K. 1956 *J. Roy. Aero. Soc.* **61**, 415–417.
OWEN, P. R. & ZIENKIEWICZ, H. K. 1957 *J. Fluid Mech.* **2**, 521–531.

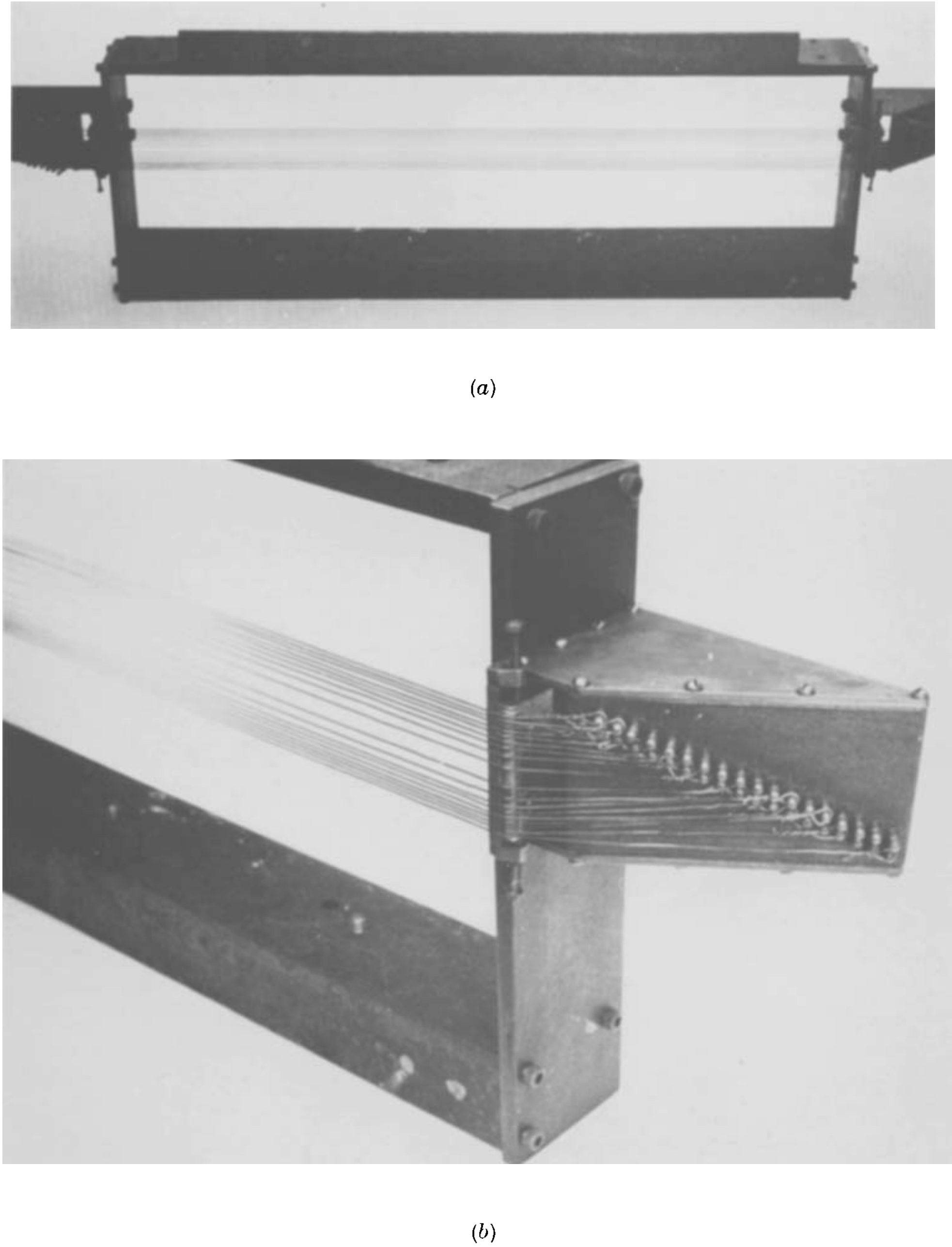


FIGURE 3. (*a*) General view of the grid arrangement. (*b*) View of the wire attachment and spacing roller.